QP CODE: 25025035

Reg No : .....

Name : .....

25025035

M.Sc DEGREE (CSS) EXAMINATION, MAY 2025

Second Semester

M Sc MATHEMATICS

CORE - ME010202 - ADVANCED TOPOLOGY

2019 ADMISSION ONWARDS

35EEC894

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that compact subset in a Hausdorff space is closed
2. Prove that a space satisfying T_2 axiom is Normal if it is compact.
3. Explain the concept of Cartesian product of the family of sets $\{X_1, X_2, \ldots, X_n\}$
4. Define a box and a wall
5. Given that each of the coordinate spaces $\{(X_i, \tau_i) : i \in I\}$ is T_2. Prove that their topological product is T_2
6. State the condition under which the evaluation function is one-to-one.
7. Prove that the union of a locally finite family of closed sets is a closed set.
8. Define a sequentially compact space with an example.
9. Let X be any set and $\mathcal{F}$ the set of all finite subsets of X. For F, G in $\mathcal{F}$, define $F \geq G$ to mean $F \supset G$. Prove that $\geq$ directs $\mathcal{F}$
10. If a net $S : D \to X$ has $x \in X$ as limit then prove that so does every subnet of S.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let X be a topological space and suppose $\{F_t : t \in Q\}$ is a family of sets in X such that

- F_t is open in X for each t in Q,
- $F_t = \emptyset$ for $t < 0$, $t \in Q$
- $F_t = X$ for $t > 1$, $t \in Q$
- $\overline{F}_s \subseteq F_t$, where $s,t \in Q, s < t$

For each x in X, define f(x)=inf {t∈Q: x ∈ F_t } .Show that f is a continuous real valued function on X and it take values in the unit interval [0,1]

12. Let A be a closed subset of a normal space X and suppose f:A→(-1,1) is a continuous function. Show that there exists a continuous function F:X→(-1,1) such that $F(x) = f(x) \forall x \in A$

13. Prove that the product of a family of closed sets is a closed set in the product topology

14. Define the following (1) I'th power of the space Y (2) Hilbert Cube (3) Cantor discontinuum

15. Let $\{f_i : X \to Y_i | i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Then prove that the corresponding evaluation function $e : X \to \prod_{i \in I} Y_i$ is open when regarded as a function from X onto $e(X)$.

16. Prove that the continuous image of a countably compact space is countably compact.

17. Let (D, ≥) is a directed set and E is an eventual subset of D and let X be a topological space.If a net S: D→ X converges to x in X iff the restriction S/E : E → X converges to x in X

18. Show that if $h, h' : X \to Y$ and $k, k' : Y \to Z$ are homotopic then prove that $k \circ h$and $k' \circ h'$ from X to Z are homotopic.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Define extension of a function

    b) Let $X = [0,1], A = (0,1]$and define $f : A \to R$ by $f(x) = sin\frac{1}{x}$. Then prove that f cannot be continuously extended to X

    c) Show that any two extensions of a continuous function $f : A \to R$ to X where A is a subset of X agrees on $\overline{A}$

21.

20. (a) Let S be a sub-base for a topological space X. Prove that X is completely regular if and only if for each $V \in S$ and for each $x \in V$, there exist a continuous function $f : X \to [0 \quad 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$

    (b)Prove that a product of topological spaces is completely regular if and only if each coordinate space is so.

    a) Suppose $\{Y_i : i \in I\}$ is an indexed family of sets, Z is a set and $\{\theta_i : Z \to Y_i | i \in I\}$ is a family of functions such that for any set X and any family $\{f_i : X \to Y_i | i \in I\}$ of functions , there exists a unique function $e : X \to Z$ satisfying $\theta_i oe = f_i$ for all $i \in I$. Prove that there exists a bijection h from Z to $\prod Y_i$ such that for each $i \in I, \theta_i = \pi_i oh$. Moreover prove that this bijection is unique.

    b) Let $\{Y_i : i \in I\}$be a family of sets , X a set and for each $i \in I, f_i : X \to Y_i$ a function. Then prove that the evaluation function is the only function from X into $\prod Y_i$ whose composition with the projection $\pi : \prod Y_i \to Y_i$ equals f_i for all $i \in I$.

22. When a net is said to converge in a topological space? Prove that a space X is T_2 if and only if no net in X can converge to two different points.

(2×5=10 weightage)