

QP CODE: 25025035



25025035

Reg No :

Name :

M.Sc DEGREE (CSS) EXAMINATION, MAY 2025

Second Semester

M Sc MATHEMATICS

CORE - ME010202 - ADVANCED TOPOLOGY

2019 ADMISSION ONWARDS

35EEC894

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight 1 each.

1. Show that compact subset in a Hausdorff space is closed
2. Prove that a space satisfying T_2 axiom is Normal if it is compact.
3. Explain the concept of Cartesian product of the family of sets $\{X_1, X_2, \dots, X_n\}$
4. Define a box and a wall
5. Given that each of the coordinate spaces $\{(X_i, \tau_i) : i \in I\}$ is T_2 . Prove that their topological product is T_2
6. State the condition under which the evaluation function is one-to-one.
7. Prove that the union of a locally finite family of closed sets is a closed set.
8. Define a sequentially compact space with an example.
9. Let X be any set and \mathcal{F} the set of all finite subsets of X . For F, G in \mathcal{F} , define $F \geq G$ to mean $F \supset G$. Prove that \geq directs \mathcal{F}
10. If a net $S : D \rightarrow X$ has $x \in X$ as limit then prove that so does every subnet of S .

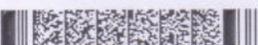
(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Let X be a topological space and suppose $\{F_t : t \in Q\}$ is a family of sets in X such that
 - F_t is open in X for each t in Q ,
 - $F_t = \emptyset$ for $t < 0, t \in Q$
 - $F_t = X$ for $t > 1, t \in Q$
 - $\overline{F_s} \subseteq F_t$, where $s, t \in Q, s < t$





For each x in X , define $f(x) = \inf \{t \in \mathbb{Q} : x \in F_t\}$. Show that f is a continuous real valued function on X and it takes values in the unit interval $[0, 1]$

12. Let A be a closed subset of a normal space X and suppose $f: A \rightarrow (-1, 1)$ is a continuous function. Show that there exists a continuous function $F: X \rightarrow (-1, 1)$ such that $F(x) = f(x) \forall x \in A$
13. Prove that the product of a family of closed sets is a closed set in the product topology
14. Define the following (1) l^p power of the space Y (2) Hilbert Cube (3) Cantor discontinuum
15. Let $\{f_i : X \rightarrow Y_i | i \in I\}$ be a family of functions which distinguishes points from closed sets in X . Then prove that the corresponding evaluation function $e : X \rightarrow \prod_{i \in I} Y_i$ is open when regarded as a function from X onto $e(X)$.
16. Prove that the continuous image of a countably compact space is countably compact.
17. Let (D, \geq) is a directed set and E is an eventual subset of D and let X be a topological space. If a net $S: D \rightarrow X$ converges to x in X iff the restriction $S/E : E \rightarrow X$ converges to x in X
18. Show that if $h, h' : X \rightarrow Y$ and $k, k' : Y \rightarrow Z$ are homotopic then prove that $k \circ h$ and $k' \circ h'$ from X to Z are homotopic.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. a) Define extension of a function
 b) Let $X = [0, 1]$, $A = (0, 1]$ and define $f : A \rightarrow \mathbb{R}$ by $f(x) = \sin \frac{1}{x}$. Then prove that f cannot be continuously extended to X
 c) Show that any two extensions of a continuous function $f : A \rightarrow \mathbb{R}$ to X where A is a subset of X agrees on \overline{A}
20. (a) Let \mathcal{S} be a sub-base for a topological space X . Prove that X is completely regular if and only if for each $V \in \mathcal{S}$ and for each $x \in V$, there exist a continuous function $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$
 (b) Prove that a product of topological spaces is completely regular if and only if each coordinate space is so.
 a) Suppose $\{Y_i : i \in I\}$ is an indexed family of sets, Z is a set and $\{\theta_i : Z \rightarrow Y_i | i \in I\}$ is a family of functions such that for any set X and any family $\{f_i : X \rightarrow Y_i | i \in I\}$ of functions, there exists a unique function $e : X \rightarrow Z$ satisfying $\theta_i \circ e = f_i$ for all $i \in I$. Prove that there exists a bijection h from Z to $\prod Y_i$ such that for each $i \in I$, $\theta_i = \pi_i \circ h$. Moreover prove that this bijection is unique.
 b) Let $\{Y_i : i \in I\}$ be a family of sets, X a set and for each $i \in I$, $f_i : X \rightarrow Y_i$ a function. Then prove that the evaluation function is the only function from X into $\prod Y_i$ whose composition with the projection $\pi : \prod Y_i \rightarrow Y_i$ equals f_i for all $i \in I$.
22. When a net is said to converge in a topological space? Prove that a space X is T_2 if and only if no net in X can converge to two different points.

(2×5=10 weightage)

